Radical Equations

If you have the variable (usually x) under the radical, we cannot solve for it until we get it out from under the radical. The way to do this is to get the expression with the radical by itself and then square both sides. When the radical is by itself, squaring it gets rid of the radical. At this level, we will wind up with either a linear equation or a quadratic equation (both of which we covered in this packet).

Example 1: $\quad 7\sqrt{11x+3} - 46 = -4$

$$+46 \qquad +46$$

$$\frac{7\sqrt{11x+3}}{7} = \frac{42}{7}$$

$\sqrt{11x+3} = 6$ *(Radical is isolated, square both sides.. this gets rid of the radical on the left side of the equation).*

$$\left(\sqrt{11x+3}\right)^2 = (6)^2$$

$$11x + 3 = 36$$
$$-3 \qquad -3$$

$$\frac{11x}{11} = \frac{33}{11}$$

$$X = 3 \quad \text{*You MUST do your check to make sure your solution is not an extraneous root.}$$

Check: $7\sqrt{11x+3} - 46 = -4 \qquad x = 3$

$$7\sqrt{11(3)+3} - 46 = -4$$
$$7\sqrt{33+3} - 46 = -4$$
$$7\sqrt{36} - 46 = -4$$
$$7(6) - 46 = -4$$
$$42 - 46 = -4$$
$$-4 = -4 \quad ✓$$

$$x = \{3\}$$

<u>Example 2:</u> $\sqrt{13x + 30} - x = 0$

$$\underline{\phantom{\sqrt{13x+30}} +x \qquad +x}$$

$$\sqrt{13x + 30} \qquad = x$$

$(\sqrt{13x + 30})^2 \quad = (x)^2$ **(*Radical is isolated, square both sides*)**

$$13x + 30 \quad = x^2$$
$$\underline{-13x \ - 30 \qquad\qquad -13x - 30}$$

$0 = x^2 - 13x - 30$ *(Factor and solve)*
$0 = \underline{(x + 2)(x - 15)}$
$x + 2 = 0 \qquad x - 15 = 0$
$\underline{\ \ -2 \ \ -2 \qquad +15 \quad +15}$
$x \quad = -2 \qquad x \quad = 15$ *(Check for extraneous roots)*

<u>*Check:*</u>
$\sqrt{13x + 30} - x = 0$

X = -2   *x = 15*

$\sqrt{13(-2) + 30} - (-2) = 0$ $\sqrt{13(15) + 30} - (15) = 0$
$\sqrt{-26 + 30} - (-2) = 0$ $\sqrt{195 + 30} - (15) \quad = 0$
$\sqrt{4} - (-2) \qquad\quad = 0$ $\sqrt{225} - (15) \qquad = 0$
$2 \ - (-2) \qquad\quad = 0$ $15 \ - \ (15) \qquad\quad = 0$

4 $\neq 0$ **X**   *0* $= 0$ ✓

$$x = \{15\}$$

Example 3: $\sqrt{12x - 20} - x = 1$

$$\underline{+ \text{ X} \qquad + \text{ X}}$$

$\sqrt{12x - 20} \qquad = x + 1$ *(Radical is isolated, square both sides)*

$(\sqrt{12x - 20})^2 \qquad = (x + 1)^2$
$(\sqrt{12x - 20})^2 \qquad = (x + 1)(x + 1)$
$12x - 20 \qquad = x^2 + 1\text{x} + 1\text{x} + 1$
$12x - 20 \qquad = x^2 + 2\text{x} + 1$
$\underline{-12\text{x} + 20 \qquad\qquad -12\text{x} + 20}$

$0 \quad = \quad \text{x}^2 - 10\text{x} + 21$ *(Factor and solve)*
$0 \qquad = \underline{(\text{x - 3})(\text{x - 7})}$
$\quad\qquad \text{x - 3} = 0 \quad \text{x} - 7 = 0$
$\qquad \underline{+3 \quad +3 \quad +7 \quad +7}$
$\quad x \quad = 3 \qquad x \quad = 7$ *(Check for extraneous roots)*

Check:

$\sqrt{12x - 20} - x = 1$

$X = 3$ $x = 7$

$\sqrt{12(3) - 20} - (3) = 1$ $\sqrt{12(7) - 20} - (7) = 1$
$\sqrt{36 - 20} \quad - (3) = 1$ $\sqrt{84 - 20} \quad - (7) \quad = 1$
$\sqrt{16} - (3) \qquad = 1$ $\sqrt{64} - (7) \qquad\quad = 1$
$4 \ - (3) \qquad = 1$ $8 \ - \ (7) \qquad\quad = 1$
$1 \qquad\qquad = 1$ ✔ $1 \qquad\qquad = 1$ ✔

$$\text{x} = \{3,7\}$$

Example 4: $\sqrt{11x + 36} + 6 = x$

$-6-6$

$\sqrt{11x + 36} = x - 6$ *(Radical is isolated, square both sides)*

$(\sqrt{11x + 36})^2 = (x - 6)^2$

$(\sqrt{11x + 36})^2 = (x - 6)(x - 6)$

$11x + 36 = x^2 -6x - 6x + 36$

$11x + 36 = x^2 - 12x + 36$

$-11x \; - 36 -11x \; - 36$

$0 = x^2 - 23x$ *(Factor and solve)*

$0 = x(x - 23)$

$x = 0 \quad x - 23 = 0$

$+23 \quad +23$

$x = 23$

Check:

$\sqrt{11x + 36} + 6 = x$

X = 0   *x = 7*

$\sqrt{11(0) + 36} + 6 = (0)$ $\sqrt{11(23) + 36} + 6 = (23)$

$\sqrt{0 + 36} + 6 = 0$ $\sqrt{253 + 36} + 6 = 23$

$\sqrt{36} + 6 = 0$ $\sqrt{289} + 6 = 23$

$6 + 6 = 0$ $17 + 6 = 23$

 *12* $ \neq 0$ **X** 23 $ = 23$ ✓

$$x = \{23\}$$

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1) $\sqrt{14x + 1} = 13$

2) $3\sqrt{8x + 9} - 24 = -3$

3) $\frac{1}{3}\sqrt{21 - 4x} + 17 = 20$

4) $\sqrt{72 - x} = x$

5) $\sqrt{-19x - 60} = x$

6) $-\sqrt{-19x - 60} = x$

7) $\sqrt{16x - 48} = x$

8) $\sqrt{12x + 4} - 3 = x$

9) $\sqrt{11x + 26} - x = 4$

10) $\sqrt{12x + 4} - 2 = x$

11) $\sqrt{8x + 1} - x = -1$

12) $\sqrt{41 - 2x} - 11 = x$

13) $\sqrt{8x + 25} - x = -5$

14) $\sqrt{-28 - 11x} = x$

15) $\sqrt{13x + 61} - 7 = x$

Quiz Review:

1) $\sqrt{7x - 6} = x$ x = {1,6}

2) $6\sqrt{6x - 2} - 55 = -7$ x = 11

3) $\frac{1}{2}\sqrt{11x + 31} + 9 = 16$ $\qquad\qquad\qquad\qquad$ x = 15

4) $\sqrt{x^2 - 48} + 2 = x$ $\qquad\qquad\qquad\qquad$ x = 13

5) $\sqrt{5x + 4} - x = -2$ $x = \{9\}$, 0 is an extraneous root

6) $\sqrt{14x - 6} - x = 3$ $x = \{3,5\}$

7) $\sqrt{10x - 1} + 4 = x$ x = {17},1 is an extraneous root

8) $\sqrt{9x + 1} - x = -1$ x = {11}, 0 is an extraneous root

9) $\frac{3}{4}\sqrt{13x - 38} - 8 = 1$ $x = 14$

10) $\sqrt{11x + 49} - 7 = x$ $x = \{-3, 0\}$